Lesson 7. Vector Functions and Space Curves

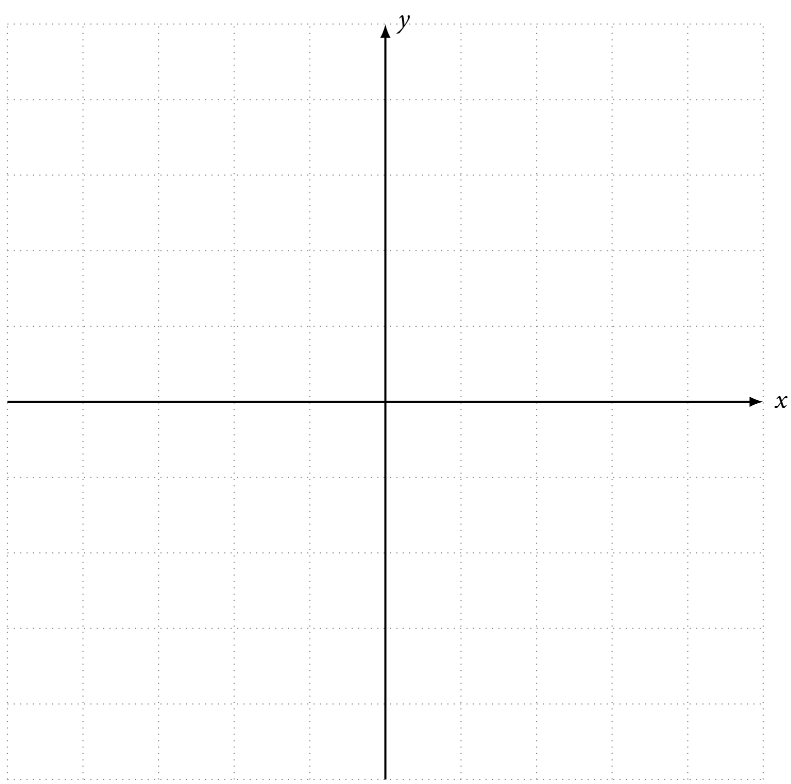
1 In this lesson...

- How do we describe curves in 3D space, especially those that are not lines?

2 Vector functions

- A **vector function**
  - takes a real number as input and
  - outputs a vector
- For example, a 3D vector function:
$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$
where $f(t)$, $g(t)$, and $h(t)$ are real-valued functions
  - $f(t)$, $g(t)$, and $h(t)$ are the **component functions** of $\vec{r}(t)$
- We can also have 2D vector functions: $\vec{r}(t) = \langle f(t), g(t) \rangle$

Example 1. Let $\vec{r}(t) = \langle t, 1 - t^2 \rangle$. Draw the vectors $\vec{r}(0)$, $\vec{r}(1)$, and $\vec{r}(2)$ with their tails starting at the origin.



3 Space curves

- Suppose f, g, h are (continuous) real-valued functions
- A **space curve** is the set of all points (x, y, z) in space that satisfy

$$x = f(t) \qquad y = g(t) \qquad z = h(t)$$

  as t varies in some interval, such as $(-\infty, +\infty)$

- Alternatively, we can describe the same curve by the vector function $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$
  - Each value of t results in the position vector $\vec{r}(t)$ of a point on the curve
- Recall the parametric equations for a line in 3D space:

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

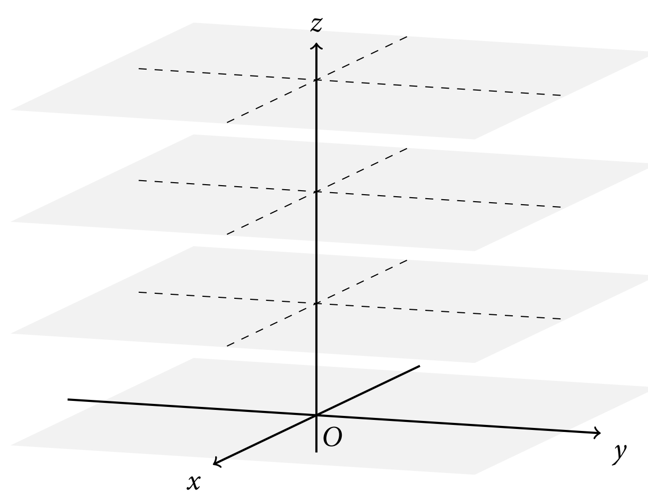
  where (x_0, y_0, z_0) is a point on the line, and $\langle a, b, c \rangle$ is the direction vector of the line

  - This fits the definition of a space curve: let

$$f(t) = x_0 + at \qquad g(t) = y_0 + bt \qquad h(t) = z_0 + ct$$

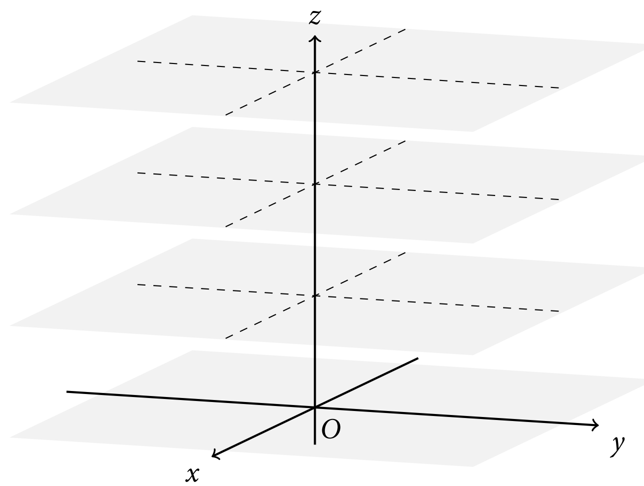
Example 2. Let $\vec{r}(t) = \langle 2, 9 - t^2, t \rangle$.

a. Evaluate $\vec{r}(t)$ at $t = 0, 1, 2, 3$.
b. Sketch the curve given by $\vec{r}(t)$.



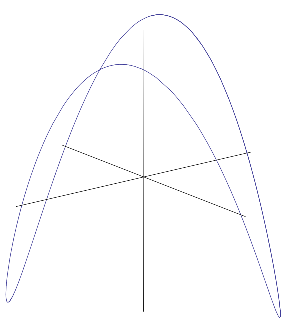
Example 3. Let $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$.

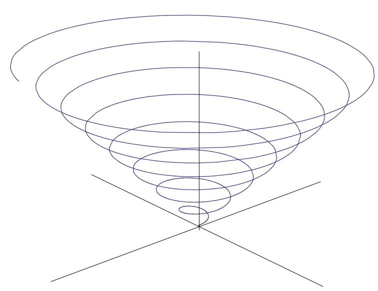
a. Evaluate $\vec{r}(t)$ at $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$.
b. Sketch the curve given by $\vec{r}(t)$.

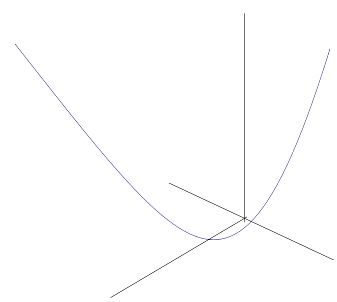


Example 4. Match the vector functions with the graphs. Give reasons for your choices.

a. $\vec{r}(t) = \langle t \cos t, t \sin t, t \rangle$
b. $\vec{r}(t) = \langle \cos t, \sin t, \sin 2t \rangle$
c. $\vec{r}(t) = \langle e^{-3t/5}, t, t^2 \rangle$







Example 5. The positions of two airplanes at time t are given by the vector functions

$$\vec{r}_1(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle \qquad \vec{r}_2(t) = \langle t, t^2, t^3 \rangle$$

Do the airplanes collide? Do their paths intersect?